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Comparison of Agility Metrics to Beck Agility Metrics Using Linear Error Theory

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Fighter agility metrics determined from measured or calculated data can be sensitive to the coordinate system used. Although simple and intuitive, agility metrics expressed in Cartesian coordinates are not always robust to variations in initial conditions and uncertainties in physical characteristics. This paper seeks to establish the relationship between form of agility equations and the measured non-poststall agility of high performance aircraft. Cartesian-coordinate-based agility metrics consisting of the time to roll through bank angle metric, time-averaged integral of pitch rate metric, and power onset/loss parameter metrics are compared to the Beck metrics, which are based in the Frenet coordinate system. Each metric is evaluated with initial condition errors and parametric uncertainties, and linear error theory is used to determine how linearly the errors propagate. Results presented demonstrate that accuracy of measured agility metrics is directly influenced by choice of coordinate system, and recommendations on which coordinate system to use with each agility metric are provided.

Nomenclature

$\boldsymbol{A}$	=	acceleration vector, ft/s^2
A_a	=	axial agility metric, ft/s^2
A_c	=	curvature agility metric, ft/s^3
A_t	=	torsional agility metric, ft/s^4
$\boldsymbol{B}$	=	burst vector, ft/s^4
C_{lp}	=	nondimensional change in rolling moment due to roll rate
$C_{l\delta a}$	=	nondimensional change in rolling moment due to aileron deflection
F_x	=	total body X-axis force, lb
F_y	=	total body Y-axis force, lb
F_z	=	total body Z-axis force, lb
g	=	gravitational constant, ft/s^2
I_{xx}	=	body X-axis moment of inertia, slug · ft^2
I_{xz}	=	body XZ-axis product of inertia, slug · ft^2
I_{yy}	=	body Y-axis moment of inertia, slug · ft^2
I_{zz}	=	body Z-axis moment of inertia, slug · ft^2
$\boldsymbol{J}$	=	jerk vector, ft/s^3
L	=	total moment about body X axis, also rolling moment, ft · lb
M	=	total moment about body Y axis, also pitching moment, ft · lb
M_a	=	axial maneuver performance metric, ft/s
M_c	=	curvature maneuver performance metric, ft/s^2
M_t	=	torsional maneuver performance metric, ft/s^3
N	=	total moment about body Z axis, also yawing moment, ft · lb
P_s	=	specific excess power, ft/s
$\dot{P}_S$	=	power onset/loss parameter, ft/s^2
p_f	=	tangential-axis angular velocity, deg/s
u_b	=	body X-axis forward velocity, ft/s
V_p	=	total velocity, ft/s
v_b	=	body Y-axis forward velocity, ft/s
v_f	=	tangential-axis forward velocity, ft/s
W	=	aircraft weight, lb
w_b	=	body Z-axis forward velocity, ft/s
τ	=	rotation angle between wind axis and Frenet axis, deg
$\boldsymbol{\Omega}_b$	=	body-axis angular velocity vector, deg/s
$\boldsymbol{\Omega}_f$	=	Frenet-axis angular velocity vector, deg/s
ω	=	binormal-axis angular velocity, deg/s

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Introduction

FIGHTER agility metrics, or simply agility metrics, are measures of merit intended to quantify and compare the combat effectiveness of fighter type aircraft.[1] Collectively, they quantify both low-speed, poststall capability and also high rate and acceleration transient maneuvering capability. Agility metrics differ from traditional steady-state measures such as maximum speed, maximum load factor, and maximum sustained turn rate, in that they are intended to describe the transient capability of the aircraft, or how the aircraft transitions from one steady-state flight condition to another.[2] This consideration is an important one because technology improvements in propulsion, thrust vectoring, and all-aspect missiles have decreased the time of engagements and increased the transient aspect of maneuvering.

Understanding the effects of nonlinearities in dynamical systems, the validity of linear systems used to analyze them, and the propagation of errors is central to the study of agility metrics. Bohacek and Jonckheere[3] investigated the structural stability of linear dynamically varying systems, or linearized models of nonlinear systems on compact sets. Smith and Valasek[4] used linear error theory to study the robustness of agility metrics to error propagation, or how measurement errors propagate with time through the nonlinear differential or algebraic agility equations. Four agility metrics were evaluated: the time to roll through bank angle metric, time-averaged integral of pitch rate metric, the power onset metric, and power loss parameter metric. Because many of the most important agility metrics can be represented by more than one equation, the question to be answered is the following: Which particular equations are robust with respect to errors and by how much? Errors cannot be avoided altogether, but if errors can be shown to propagate linearly with time through a particular equation, then agility results obtained with that equation might be considered acceptable. On the other hand, if it can be shown that errors propagate nonlinearly with time through an equation, then agility results obtained with that equation could realistically be regarded with skepticism or discarded altogether. Additionally, equations can be expressed in numerous coordinate systems; therefore, another question is as follows: Which form of the equation (with respect to coordinate system) is more robust with respect to errors and by how much?

This paper compares the error sensitivity and propagation of four non-poststall agility metrics expressed in Cartesian coordinates[1] to the error sensitivity of agility metrics expressed in Frenet coordinates for the same input data. Starting with Frenet-axis agility metrics parameterized in terms of Cartesian body-axis accelerations and rates, a new parameterization is derived in terms of Cartesian body-axis forces, moments, and moments of inertia. Linear error theory is then used to perform an error propagation sensitivity analysis on each agility metric, using a nonlinear, non-real-time, six-degree-of-freedom simulation of an uninhabited air vehicle (UAV). Finally, results are presented that directly compare the robustness of the different parameterizations to errors.

Agility Metrics

Aircraft agility describes the transient capability of the aircraft, or how responsively the aircraft transitions from one steady state to another. Agility metrics have been proposed to quantify this multidimensional behavior. Agility metrics more completely define maneuvering performance by quantifying the maneuvers that dynamically change flight conditions than steady-state or point performance measures. These metrics measure the effects engine characteristics such as engine spool time and maximum thrust, roll performance while operating at high but non-poststall angles of attack, thrust vectoring, and pitching and load factor capabilities of the aircraft.[5]

Later work by Beck and Cord[6] followed a strictly mathematical approach to the definitions of maneuverability and agility of Ref. 5. In their work, Beck and Cord[6] have the following definitions:

1) Maneuverability is a measure of the ability of an aircraft to achieve and transition between steady maneuvers.

2) Maneuver performance is a measure of the aircraft's steady capability.

3) Agility is measure of the aircraft's capability to transition between states.

These terms are then related to the derived velocity, acceleration, jerk, and burst motion vectors in the Frenet axis system, shown in Fig. 1. The Frenet reference frame is a noninertial system that osculates the trajectory of the mass center, in which the X_F axis is tangential to the flight path, the principal normal axis Y_F is perpendicular to X_F and lies in the maneuver plane formed by the velocity vector and the total force vector, and the Z_F binormal axis is perpendicular to the maneuver ($X_F Y_F$) plane. It is oriented in a right-handed coordinate system, with the quantities $\boldsymbol{t}$, $\boldsymbol{n}$, and $\boldsymbol{b}$ representing unit vectors along the X_F, Y_F, and Z_F axes, respectively.[6]

The maneuver performance parameters are characterized by the translational rate and rotational rates, and the agility components are specified by the maneuver performance parameters and their derivatives. The velocity of the aircraft expressed in the Frenet system is given by

$$\boldsymbol{V} = v\boldsymbol{t} \tag{1}$$

where v is the speed of the aircraft. The rotation of the Frenet axis system with respect to the inertial axis system is defined by the

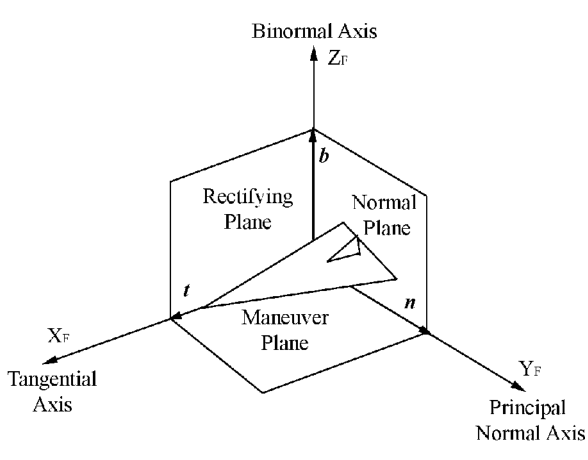

Fig. 1 Components of Frenet axis system.

rotation vector,

$$\boldsymbol{\Omega}_f = p_f\boldsymbol{t} + q_f\boldsymbol{n} + \omega\boldsymbol{b} \tag{2}$$

The first component of the rotation vector, p_f, is the maneuver-plane roll rate, and ω is the turn rate in the maneuver plane. The rotation component about the normal axis, q_f, is identically zero as there is no component of force in the binormal direction.[6] Using Eq. (2) and differentiating Eq. (1) gives the acceleration vector

$$\boldsymbol{A} = \dot{v}\boldsymbol{t} + v\omega\boldsymbol{n} \tag{3}$$

Differentiating again gives the jerk vector

$$\boldsymbol{J} = (\ddot{v} - v\omega^2)\boldsymbol{t} + (2\dot{v}\omega + v\dot{\omega})\boldsymbol{n} + (v\omega p_f)\boldsymbol{b} \tag{4}$$

Finally, taking the derivative of the jerk vector gives the burst vector,

$$\boldsymbol{B} = (\dddot{v} - 3\dot{v}\omega^2 - 3v\omega\dot{\omega})\boldsymbol{t} + \left(3\ddot{v}\omega + 3\dot{v}\dot{\omega} + v\ddot{\omega} - v\omega^3 - v\omega p_f^2\right)\boldsymbol{n} + (3\dot{v}\omega p_f + 2v\dot{\omega}p_f + v\omega\dot{p}_f)\boldsymbol{b} \tag{5}$$

The components of the motion vectors are related to the maneuver performance metrics and the agility metrics. The resulting Beck maneuver performance metrics (see Ref. 6) are as follows.

Axial maneuver performance:

$$M_a = v \tag{6}$$

Curvature maneuver performance:

$$M_c = v\omega \tag{7}$$

Torsional maneuver performance:

$$M_t = v\omega p_f \tag{8}$$

The Beck agility metrics (see Ref. 6) are as follows.

Axial agility:

$$A_a = \dot{v} \tag{9}$$

Curvature agility:

$$A_c = 2\dot{v}\omega + v\dot{\omega} \tag{10}$$

Torsional agility:

$$A_t = 3\dot{v}\omega p_f + 2v\dot{\omega}p_f + v\omega\dot{p}_f \tag{11}$$

McKeehen and Cord[7] investigated the application of these maneuver performance and agility metrics, which they labeled as the Beck metrics, to aircraft data. Two methods were used to calculate the Beck axial, curvature, and torsional metrics. The first transformed vectors defined relative to a body-fixed axis system to the Frenet axis system, using a coordinate transformation to calculate the parameters of the Beck metrics. The second method used the inertial position vector and its derivatives and utilized the vector analytical/differential geometric quantities of curvature κ and torsion τ to calculate the parameters of the Beck metrics.

Calculation of the Beck metrics using the first method is outlined here. Consider an axis system with mutually orthogonal axes labeled X_b, Y_b, and Z_b originating from the center of gravity of an aircraft and rigidly attached to it with the orientation shown in Fig. 2. This coordinate system will be called the body-axis system. Rotating X_b about Y_b through the angle of attack to the total aircraft velocity vector $\boldsymbol{V}_p$ results in a coordinate system called stability axes, X_s, Y_s, and Z_s, originating from the center of gravity. In stability axes X_s axis lies along the projection of $\boldsymbol{V}_p$ onto the plane of symmetry, and the Z_s axis is perpendicular to the X_s axis and lies in the plane of symmetry pointing in the downward direction when the normal attitude of the aircraft is considered. The Y_s axis is such that $X_s Y_s Z_s$ form a right-handed set. The wind axes (X_w, Y_w, Z_w) are obtained

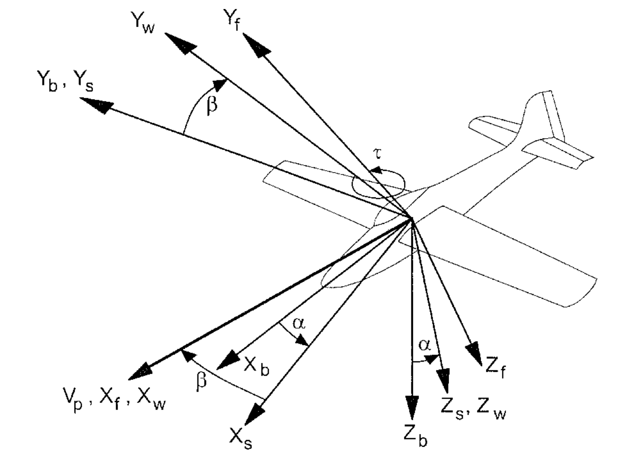

Fig. 2 Relationships between body, stability, wind, and Frenet axis systems.

by rotating about Z_s through the sideslip angle. In this axis system X_w lies along $\boldsymbol{V}_p$ and is, therefore, tangential to the flight path in the direction of motion (Fig. 2). The Z_w axis is the same as the Z_s axis, and Y_w is perpendicular to both X_w and Z_w so that $X_w Y_w Z_w$ form a right-handed set. The total transformation between the body axes and the Frenet axes is obtained through a series of rotations, with the stability and wind axes as intermediate steps. The sequence of rotations is a 2–3–1 rotation from the body axes about Y_b through α, about Z_s through β, and about X_w through τ, which is defined as the angle between the positive Y_w axis and the positive Y_f axis (Fig. 2). Note that angle τ is not related to the torsion parameter of the second method mentioned earlier. The body angular velocity vector is defined as

$$\boldsymbol{\Omega}_b = p\boldsymbol{i}_b + q\boldsymbol{j}_b + r\boldsymbol{k}_b \tag{12}$$

and by using the transformation and the relative velocity equation, the Frenet angular velocity components are

$$p_f = p\cos\alpha\cos\beta + q\sin\beta + r\sin\alpha\cos\beta - \dot{\tau} + \dot{\alpha}\sin\beta \tag{13}$$

$$\begin{aligned} q_f = {} & -p(\cos\tau\cos\alpha\sin\beta + \sin\tau\sin\alpha) \\ & + r(\sin\tau\cos\alpha - \cos\tau\sin\alpha\sin\beta) + q(\cos\tau\cos\beta) \\ & - \dot{\beta}\sin\tau + \dot{\alpha}\cos\tau\cos\beta \end{aligned} \tag{14}$$

$$\begin{aligned} \omega = {} & |p(\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha) \\ & + r(\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha) \\ & - q\sin\tau\cos\beta - \dot{\beta}\cos\tau - \dot{\alpha}\sin\tau\cos\beta| \end{aligned} \tag{15}$$

In Eq. (15), the absolute value signs are included based on the relation $\omega = v\kappa$, where v is the speed of the aircraft and flight-path curvature $\kappa \geq 0$ by definition.[7] Therefore, ω is strictly positive. The angle τ can be calculated using Eq. (14) and the fact that $q_f = 0$. The resulting equation for τ in terms of the body parameters is

$$\tau = \tan^{-1}\left[\frac{(p\cos\alpha + r\sin\alpha)\sin\beta - (\dot{\alpha} + q)\cos\beta}{r\cos\alpha - p\sin\alpha - \dot{\beta}}\right] \tag{16}$$

Equations (10) and (11) require the first derivative of p_f and ω. The resulting derivatives are

$$\begin{aligned} \dot{p}_f = {} & \dot{p}\cos\alpha\cos\beta + \dot{q}\sin\beta + \dot{r}\sin\alpha\cos\beta - p(\dot{\beta}\cos\alpha\sin\beta \\ & + \dot{\alpha}\sin\alpha\cos\beta) + q\dot{\beta}\cos\beta - r(\dot{\beta}\sin\alpha\sin\beta - \dot{\alpha}\cos\alpha\cos\beta) \\ & + \ddot{\alpha}\sin\beta + \dot{\alpha}\dot{\beta}\cos\beta - \ddot{\tau} \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{\omega} = {} & \{\dot{p}(\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha) - \dot{q}\sin\tau\cos\beta \\ & + \dot{r}(\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha) + p[\dot{\beta}\sin\tau\cos\alpha\cos\beta \\ & - \dot{\alpha}(\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha) + \dot{\tau}(\cos\tau\cos\alpha\sin\beta \\ & + \sin\tau\sin\alpha)] + q(\dot{\beta}\sin\tau\sin\beta - \dot{\tau}\cos\tau\cos\beta) \\ & + r[\dot{\beta}\sin\tau\sin\alpha\cos\beta + \dot{\alpha}(\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha) \\ & + \dot{\tau}(\cos\tau\sin\alpha\sin\beta - \sin\tau\cos\alpha)] + \dot{\beta}\dot{\tau}\sin\tau - \ddot{\beta}\cos\tau \\ & - \dot{\alpha}\dot{\tau}\cos\tau\cos\beta - \ddot{\alpha}\sin\tau\cos\beta + \dot{\alpha}\dot{\beta}\sin\tau\sin\beta\}|(\omega|/\omega) \end{aligned} \tag{18}$$

Using Eqs. (13–18) with velocity and velocity acceleration, the Beck maneuver performance and agility metrics can be calculated from simulation or flight data (see Ref. 7).

The Beck maneuver performance and agility metrics, Eqs. (6–11), are expressed in terms of the Frenet system velocity, linear accelerations, angular accelerations, and angular rates. Using McKeehen and Cord's[7] first method to transform to the Frenet coordinate system from the body axes, the Frenet-based rates and accelerations are expressed in terms of the body-axis rates and accelerations. These expressions are Eqs. (13–18). The body-axis linear accelerations can be expressed in terms of the total body-axis forces F_x, F_y, and F_z (sum of aerodynamic and propulsive contributions); body-axis linear velocities u_b, v_b, and w_b; and angular rates p, q, and r. The body-axis angular accelerations can be expressed in terms of the total body-axis moments L, M, and N (sum of aerodynamic and propulsive contributions); moments of inertia I_{xx}, I_{yy}, I_{zz}, and I_{xz}; and the body-axis angular rates. The Beck maneuver performance and agility metrics are similarly calculated, but the Beck-axis terms $\dot{v}$, $\dot{p}_f$, and $\dot{\omega}$ are now parameterized as follows:

$$\begin{aligned} \dot{v} = {} & (1/m)[\cos\beta\cos\alpha F_x + \sin\beta F_y + \cos\beta\sin\alpha F_z] + g[-\cos\beta\cos\alpha\sin\Theta + \sin\beta\sin\Phi\cos\Theta + \cos\beta\sin\alpha\cos\Phi\cos\Theta] \\ & + \cos\beta\cos\alpha(u_b r - w_b q) + \sin\beta(-u_b r + w_b p) + \cos\beta\sin\alpha(u_b q - v_b p) \end{aligned} \tag{19}$$

$$\begin{aligned} \dot{p}_f = {} & \left(\frac{I_{xx}\sin\alpha\cos\beta + I_{zz}\cos\alpha\cos\beta}{I_{xx}I_{zz} - I_{xz}^2}\right)L + \left(\frac{\sin\beta}{I_{yy}}\right)M + \left\{\frac{I_{xz}\cos\alpha\cos\beta}{I_{xx}I_{zz} - I_{xz}^2} + \left[\frac{1}{I_{zz}} + \frac{I_{xz}^2}{I_{zz}\left(I_{xx}I_{zz} - I_{xz}^2\right)}\right]\sin\alpha\cos\beta\right\}N \\ & + \left[-\frac{\left(-I_{zz}I_{yy} + I_{zz}^2 + I_{xz}^2\right)qr - (-I_{xz}I_{xx} + I_{xz}I_{yy} - I_{xz}I_{zz})pq}{I_{xx}I_{zz} - I_{xz}^2}\right]\cos\alpha\cos\beta - r(\dot{\beta}\sin\alpha\sin\beta - \dot{\alpha}\cos\alpha\cos\beta) \\ & - p(\dot{\beta}\cos\alpha\sin\beta - \dot{\alpha}\sin\alpha\cos\beta) + \left[\frac{(I_{zz} - I_{xx})pr - I_{xz}(p^2 - r^2)}{I_{yy}}\right]\sin\beta + q\dot{\beta}\cos\beta + \left\{\frac{I_{xz}}{I_{zz}}\left(-\frac{-I_{zz}I_{yy} + I_{zz}^2 + I_{xz}^2}{I_{xx}I_{zz} - I_{xz}^2} - 1\right)qr \right. \\ & \left. + \left[\frac{I_{xx} - I_{yy}}{I_{zz}} + \frac{I_{xz}(-I_{xz}I_{xx} + I_{xz}I_{yy} - I_{xz}I_{zz})}{I_{zz}\left(I_{xx}I_{zz} - I_{xz}^2\right)}\right]pq\right\}\cos\alpha\cos\beta + \ddot{\alpha}\sin\beta + \dot{\alpha}\dot{\beta}\cos\beta - \ddot{\tau} \end{aligned} \tag{20}$$

$$\dot{\omega} = \left[\frac{I_{zz}(\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha) + I_{xz}(\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha)}{I_{xx}I_{zz} - I_{xz}^2}\right]L - \left(\frac{\sin\tau\cos\beta}{I_{yy}}\right)M$$
$$+ \left\{\frac{I_{xz}(\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha}{I_{xx}I_{zz} - I_{xz}^2} + \left[\frac{1}{I_{zz}} + \frac{I_{xz}^2}{I_{zz}\left(I_{xx}I_{zz} - I_{xz}^2\right)}\right](\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha)\right\}N$$
$$+ \left[-\frac{\left(-I_{zz}I_{yy} + I_{zz}^2 + I_{xz}^2\right)qr - (-I_{xz}I_{xx} + I_{xz}I_{yy} - I_{xz}I_{zz})pq}{I_{xx}I_{zz} - I_{xz}^2}\right](\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha)$$
$$+ r[\dot{\beta}\sin\tau\sin\alpha\cos\beta + \dot{\alpha}(\sin\tau\cos\alpha\sin\beta - \cos\tau\sin\alpha) + \dot{\tau}(\cos\tau\sin\alpha\sin\beta - \sin\tau\cos\alpha)]$$
$$+ p[\dot{\beta}\sin\tau\cos\alpha\cos\beta - \dot{\alpha}(\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha) + \dot{\tau}(\cos\tau\cos\alpha\sin\beta + \sin\tau\sin\alpha)]$$
$$- \left[\frac{(I_{zz} - I_{xx})pr - I_{xz}(p^2 - r^2)}{I_{yy}}\right]\sin\tau\cos\beta + q(\dot{\beta}\sin\tau\sin\beta - \dot{\tau}\cos\tau\cos\beta)$$
$$+ \left\{\frac{I_{xz}}{I_{zz}}\left(-\frac{-I_{zz}I_{yy} + I_{zz}^2 + I_{xz}^2}{I_{xx}I_{zz} - I_{xz}^2} - 1\right)qr + \left[\frac{I_{xx} - I_{yy}}{I_{zz}} + \frac{I_{xz}(-I_{xz}I_{xx} + I_{xz}I_{yy} - I_{xz}I_{zz})}{I_{zz}\left(I_{xx}I_{zz} - I_{xz}^2\right)}\right]pq\right\}(\sin\tau\sin\alpha\sin\beta + \cos\tau\cos\alpha)$$
$$+ \dot{\beta}\dot{\tau}\sin\tau - \ddot{\beta}\cos\tau + \dot{\alpha}\dot{\beta}\sin\beta\sin\tau - \dot{\alpha}\dot{\tau}\cos\tau\cos\beta - \ddot{\alpha}\sin\tau\cos\beta \tag{21}$$

Linear Error Theory

Linear error theory is a method of error propagation and of measuring the validity of linearity assumptions for nonlinear dynamic systems. It applies to both algebraic and dynamic systems. The severity of the nonlinearity can be measured by the use of a dynamic nonlinearity index, which Junkins defines in Ref. 8 as

$$\nu(t, t_0) \equiv \sup_i \frac{\left\| \Phi_i(t, t_0) - \Phi_{x_0}(t, t_0) \right\|_2}{\left\| \Phi_{x_0}(t, t_0) \right\|_2} \tag{22}$$

It represents the error between transition matrices $\Phi_{x_0}(t, t_0)$, which are linearizations of the nonlinear system around a prespecified trajectory of the nonlinear system. It depends on the distribution of the initial state x_0 centered on the expected value, from initial time t_0 to current time t, normalized by the transition matrix of the reference trajectory. The transition matrix satisfies

$$\dot{\Phi}(t, t_0) = J\Phi(t, t_0)$$

subject to

$$\Phi(t, t_0) = I$$

where J is the Jacobian matrix obtained by partial differentiation of the dynamic equations, evaluated about the reference trajectory. Here, the sup( ) operator extracts the maximum value of ( ) over the range of extreme (3σ) trajectories sampled at time t on the trajectory $x_i(t)$, which ensue from a 3σ surface representing a worst-case finite sample of initial conditions centered on x_0. The $\|(\ )\|_2$ denotes the Frobenious norm of a matrix. Because $\upsilon(t, t_0)$ is measured over the bundle of trajectories that ensue from a family of lower probability initial condition errors (3σ), it is felt to be a conservative measure of nonlinearity. The value of υ can range from zero (linear) to greater than one (very nonlinear). The uncertainties can be assumed to propagate linearly if $\upsilon < 10^{-2}$. If this assumption is true, standard linear approximations and methods can be used. If the nonlinearity index is larger than this value, the linear approximations are not valid, and other nonlinear methods must be used.

Numerical Evaluation of Sensitivity to Initial Conditions and Coordinate System

Four different fighter agility metrics (Table 1), each expressed in terms of the Beck maneuver performance metrics [Eq. (6–8)] and Beck agility metrics [Eq. (9–11)] are evaluated. Nominal and perturbation results are plotted for each test case, and the nonlinearity index from Ref. 4 corresponding to each test case is also included for comparison. Only the most significant graphical results are presented here for the purpose of demonstrating the method; complete graphical results of all Beck metrics for each of the four test cases are contained in Ref. 9. All results were generated using the UCAV6 simulation.[3] It is a nonlinear, non-real-time, six-degree-of-freedom simulation program of an approximately 60% scale AV-8A Harrier that has been modeled as an unmanned air vehicle (UAV) by removing all pilot-specific items and adjusting the weights and inertias accordingly. It contains steady, linear aerodynamic data in table lookup form, for angles of attack up to 35 deg. This is valid for the non-poststall results presented here. It also contains an engine dynamics model with table lookup thrust data and actuator dynamics together with rate and position limiting.

Table 1 Test case definitions

Test case	Agility metric
1	Time to roll through bank angle
2	Time-averaged integral of pitch rate
3	Power onset parameter
4	Power loss parameter

Test Case 1: Time to Roll Through Bank Angle

The nominal initial state is an altitude of 1000 ft, a velocity of 500 ft/s, and an aileron input command of 24 deg at 1.0 s. The initial condition variation is a velocity of ±50 ft/s. Parametric variations are $\Delta C_{lp} \pm 15\%$, $\Delta C_{l\delta a} \pm 25\%$, and $\Delta I_{xx} \pm 10\%$. The runs labeled variation (−) in Figs. 3–15 represent the combination of a −50-ft/s velocity error, −25% error in $C_{l\delta a}$, −10% error in I_{xx}, and 15% variation in C_{lp}. The runs labeled variation (+) are the opposite in sign combination of the same variations. The axial maneuver performance and axial agility metrics exhibited only very slight variations from the nominal for all of the initial condition errors tested and are not shown. This is expected for a lateral maneuver because the Beck axial maneuver performance metric is simply velocity [Eq. (6)], and the Beck axial agility metric is simply acceleration [Eq. (9)]. The curvature maneuver performance and curvature agility metric time histories presented in Fig. 3. Variations in the curvature maneuver performance metric for both positive and negative variations grow to almost a 50 ft/s^2 difference from the nominal. The curvature agility metric shows a large variation from the nominal result at time equals 0.9 s as the maneuver is initiated, but the variation is less than 10 ft/s^3 thereafter. Torsional maneuver performance and agility are presented in Fig. 4. The small spikes at time equals 0.9, 1.1, and 1.2 s are due to nonlinearities in calculating the angle τ. Variations

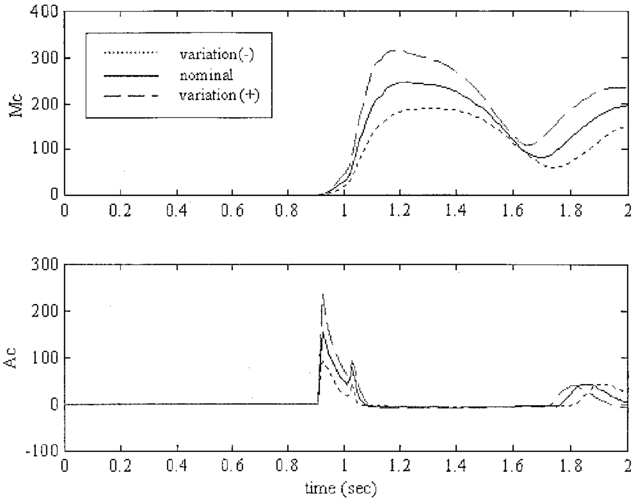

Fig. 3 Curvature maneuver performance and curvature agility time histories, test case 1.

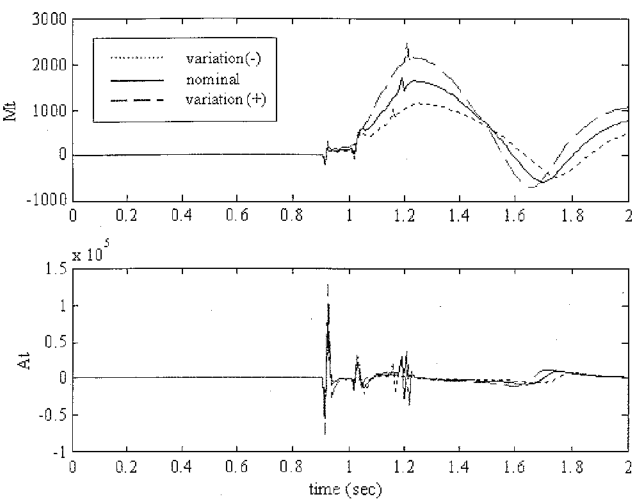

Fig. 4 Torsional maneuver performance and torsional agility time histories, test case 1.

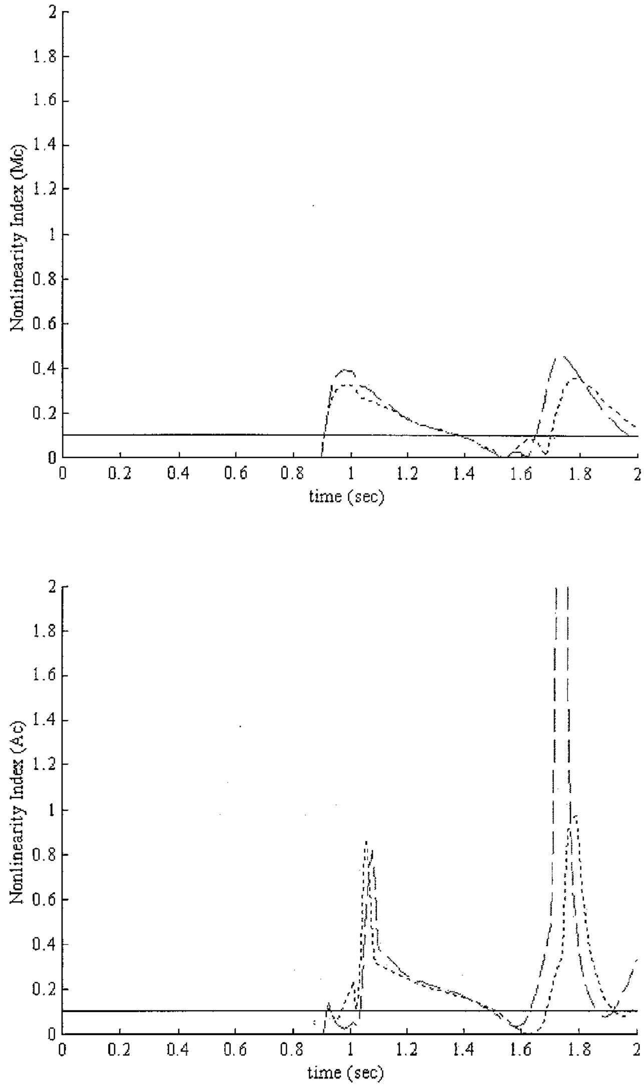

Fig. 5 Nonlinearity index for curvature maneuver performance and curvature agility, test case 1: - - - -, variation (-) and – – –, variation (+).

in the torsional maneuver performance metric for all three runs are significant, as the maximum difference is almost 500 ft/s^3 at time equals 1.25 s. The spikes in the torsional agility metric are again due to nonlinearities in the angle τ and the calculation of $\dot{\tau}$. Although variations in the torsional agility metric appear to be small due to the plot scale, variations between the positive and negative variation runs are at least 2000 ft/s^4 for the majority of the maneuver.

The nonlinearity index was calculated for each Beck maneuver performance and agility metric of test case 1 . The nonlinearity index for both the axial maneuver performance and axial agility metrics (not shown) are nearly zero for the entire time of the maneuver, indicating a linear propagation of initial condition errors in velocity, parametric uncertainty in x-axis moment of inertia, roll damping coefficient, and aileron control effectiveness. Figure 5 shows that the curvature maneuver performance metric is weakly nonlinear to initial condition errors because the nonlinearity index reaches values over 0.4. The first rise in nonlinearity is due to the divergence of the variation (−) and variation (+) maneuver plane turn rates from the nominal result during time equals 0.9–1.1 s. This can be seen in the time histories of the curvature maneuver performance metric in Fig. 3. As the variation (−) and variation (+) trajectories converge to the nominal from time equals 1.1 to 1.6 s, the nonlinearity index for both variations decrease in value. This pattern is repeated again from time equals 1.6 to 2.0 s. The nonlinearity index time histories for the curvature agility metric show the propagation of initial condition errors to be strongly nonlinear. The primary cause is the divergence and convergence of the values of the maneuver plane turn rate and turn acceleration parameters from the nominal values. The nonlinearity index for both the torsional maneuver performance metric and torsional agility metric are not shown because they are strongly nonlinear immediately on introduction of the input, and remain so thereafter. This behavior is due to a combination of the nonlinearity in τ and $\dot{\tau}$, the behavior of the maneuver plane turn rate and turn acceleration, and the nonlinearity in the maneuver plane roll rate and roll acceleration. For comparison, the nonlinearity index time histories of variation (−) and variation (+) for the time to roll through bank angle metric in Cartesian coordinates (see Ref. 9) are presented in Fig. 6. When the magnitudes in Fig. 6 are compared to the magnitudes in Fig. 5, initial condition errors and uncertainties propagate more nonlinearly in the Beck metrics than the time to roll through bank angle metric in Cartesian coordinates.

In summary, for the same lateral maneuver, the Beck axial maneuver performance and agility metrics, together with the curvature agility metric, showed the least sensitivity to initial condition errors. In terms of error propagation, the curvature maneuver performance is weakly nonlinear to initial condition errors, and the axial maneuver performance and agility metrics are linear to them. All of the other Beck maneuver performance and agility metrics are strongly nonlinear to initial condition errors. When initial condition uncertainties and error propagation are considered, the time to roll through bank angle metric in Cartesian coordinates is judged to be less sensitive overall.

Test Case 2: Time Averaged Integral of Pitch Rate

The nominal condition is an altitude of 1000 ft and a velocity of 500 ft/s. Variations in pitch rate are introduced at time equals 0.38 s.

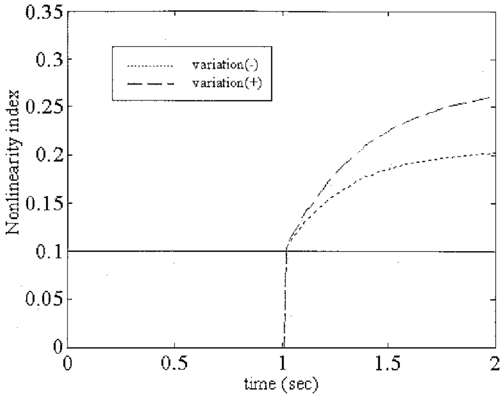

Fig. 6 Nonlinearity index for Cartesian time to roll through bank angle metric, test case 1.

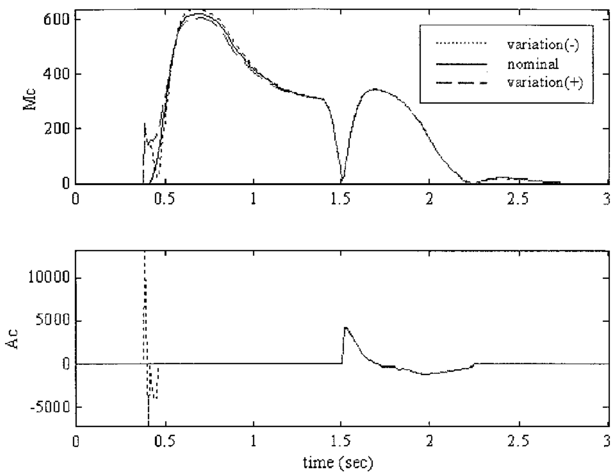

Fig. 7 Curvature maneuver performance and curvature agility time histories, test case 2.

Variation (+) is a pitch rate of 10 deg/s, and variation (−) is a pitch rate of −10 deg/s. The input enters at 0.5 s as maximum pitch up, hold for 1 s, and then pitch down to zero load factor at 1.5 s.

The metrics of interest for this purely pitching maneuver are the curvature maneuver performance and curvature agility metrics (Fig. 7). Curvature maneuver performance shows a slight variation when the input is introduced and also when the value of maximum pitch rate is achieved. The curvature agility metric is related to the pitch rate accelerations, and likewise exhibits only a small variation near time equals 0.5 s. Figure 8 shows the curvature maneuver performance metric to be linear to initial condition errors. However, variation (−) and variation (+) for the curvature agility metric increase to a value over 2 during the initial phase of the maneuver, where the pitch command is introduced. The primary cause of the nonlinearity is the nonlinearity in τ and $\dot{\tau}$, and the variation in the maneuver plane turn rate and turn acceleration parameters from the nominal case values. After the initial rise, the nonlinearity index for both variation cases is nearly zero.

Figure 9 shows the variation (−) and variation (+) results for the time-averaged integral of pitch rate metric in Cartesian coordinates (see Ref. 9). When the magnitudes are compared to those in Fig. 8, for the same maneuver, the Beck curvature maneuver performance is nearly linear to initial condition errors in pitch rate. The Beck curvature agility metric is nearly identical to the Cartesian coordinate version, being linear except during the initial phase.

In summary, for measuring the time-averaged integral of pitch rate, the Beck curvature maneuver performance metric showed the least sensitivity to initial condition errors. In terms of error propagation, the Beck curvature maneuver performance metric is nearly linear to initial condition errors. When initial condition uncertainties and error propagation are considered, the Beck curvature maneuver performance metric is judged to be less sensitive overall.

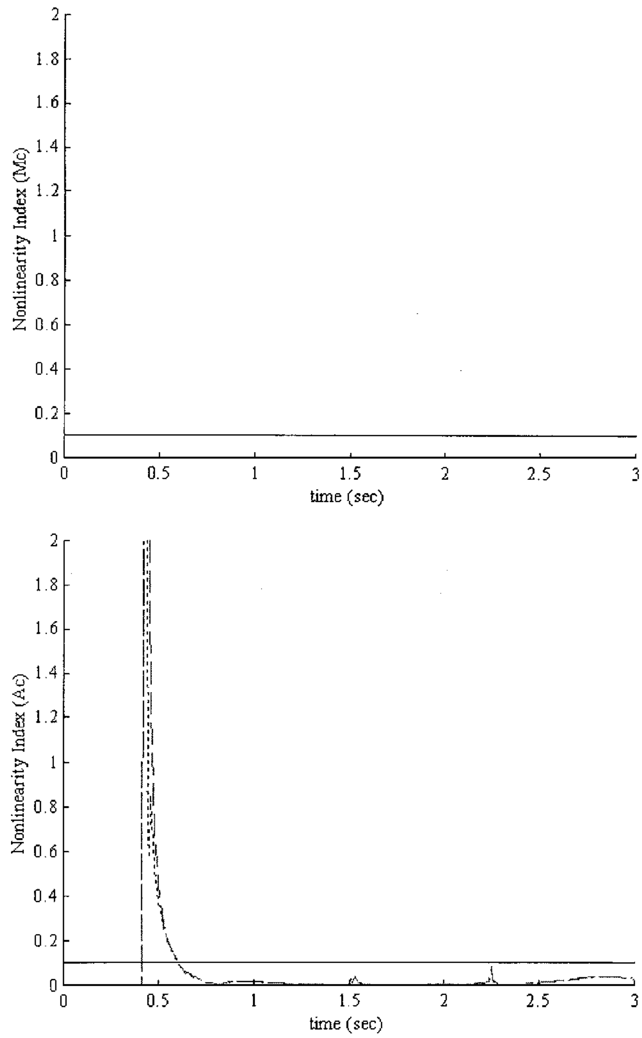

Fig. 8 Nonlinearity index for curvature maneuver performance and curvature agility, test case 2: - - - -, variation (-) and – – –, variation (+).

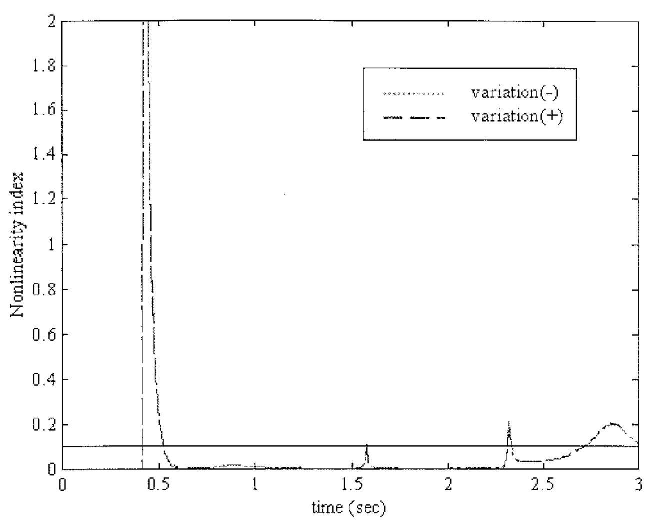

Fig. 9 Nonlinearity index for Cartesian time-averaged integral of pitch rate metric, test case 2.

Test Case 3: Power Onset Parameter

The nominal initial state is an altitude of 1000 ft and a velocity of 500 ft/s. The speed brake is deployed at time equals 0.1 s and retracted at time equals 3.5 s. Full throttle is commanded at time equals 3.5 s and held. For initial condition variations, variation (−) is a −50-ft/s velocity error, −10% thrust error, and 10% drag error. Variation (+) is the opposite in sign combination of the same variations. Because the maneuver exhibits no maneuver plane roll

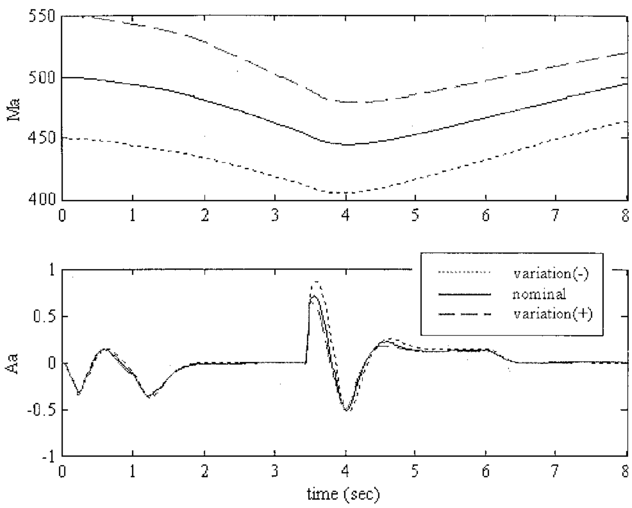

Fig. 10 Axial maneuver performance and axial agility time histories, test case 3.

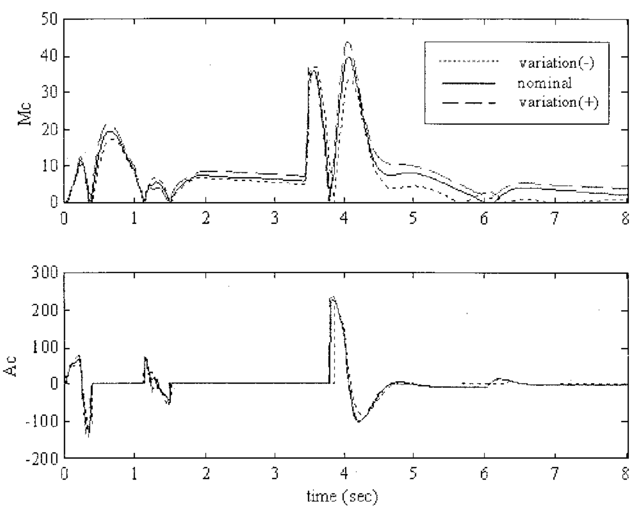

Fig. 11 Curvature maneuver performance and curvature agility time histories, test case 3.

rate, the torsional maneuver performance and agility metrics are not shown.

Figure 10 shows the axial maneuver performance metric exhibiting a wide variation due to the initial condition velocity error. In comparison the axial agility metric exhibits almost no variation. A similar trend is seen for the curvature maneuver performance and curvature agility metric time histories in Fig. 11. The maneuver performance metric oscillates as the maneuver plane turn rate oscillates, due to the pitch response induced from the speed brake extension and retraction. Variations in the curvature agility metric are very small at less than 10 ft/s^3.

The nonlinearity index for the axial maneuver performance metric is zero for the entire time of the maneuver and is not shown. This indicates that the metric is linear due to initial condition errors in velocity, thrust, and drag, as well as parametric uncertainty in drag. The nonlinearity index for the axial agility metric, shown in Fig. 12, shows it to be weakly nonlinear. The curvature maneuver performance and curvature agility metrics are strongly nonlinear, reaching nonlinearity index values well over 2.0 throughout the maneuver and are therefore not shown. The major effect is nonlinearity in the maneuver plane turn rate and in the angle τ. Figure 13 shows the variation $(-)$ and variation $(+)$ results for the power onset parameter in Cartesian coordinates (see Ref. 9). When the magnitudes are compared to those in Fig. 12, for the same maneuver the Beck axial agility metric is much more linear to initial condition errors.

In summary, for measuring the power onset parameter, the Beck axial agility and curvature agility metrics showed the least sensitivity to initial condition errors. In terms of error propagation, both the Beck axial maneuver performance metric and axial agility metric are nearly linear to initial condition errors, especially when compared to the Cartesian coordinate version of the metric. When initial condition uncertainties and error propagation are considered, the Beck axial agility metric is judged to be less sensitive overall.

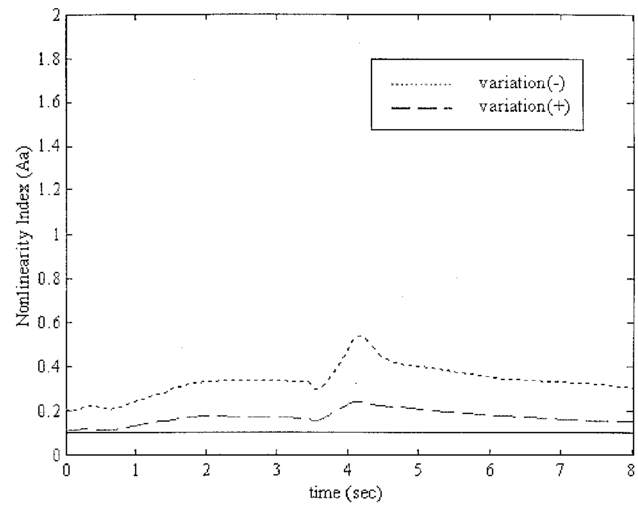

Fig. 12 Nonlinearity index for axial agility, test case 3.

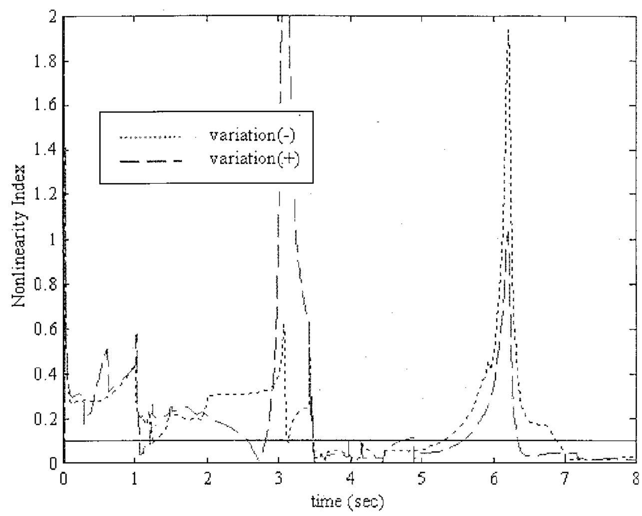

Fig. 13 Nonlinearity index for Cartesian power onset parameter metric, test case 3.

Test Case 4: Power Loss Parameter

The power loss metric is tested at the same nominal conditions, inputs, initial condition errors, and parametric uncertainties as used for test case 3. Results for the axial maneuver performance, axial agility metric, curvature maneuver performance, and curvature agility metric showed the same basic trends as for the power onset parameter of case 3. The only notable difference is for the nonlinearity index of the axial agility metric (Fig. 13), which is flatter and has a maximum value only one-third of the power onset parameter. Nonlinearity indices for the curvature maneuver performance metric and curvature agility metric are again strongly nonlinear, reaching values over 2.0 more than once during the maneuver and are not shown. Figure 14 shows the variation $(-)$ and variation $(+)$ results for the power loss parameter in Cartesian coordinates (see Ref. 9). When the magnitudes are compared to those in Fig. 13, for the same maneuver the Beck axial agility metric is clearly more linear to initial condition errors.

In summary, for measuring the power loss parameter, the Beck axial agility and curvature agility metrics showed the least sensitivity to initial condition errors. In terms of error propagation, both the Beck axial maneuver performance metric and axial agility metric are

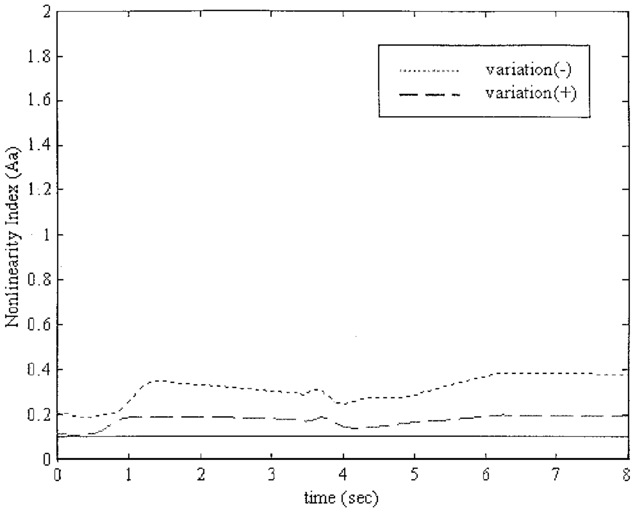

Fig. 14 Nonlinearity index for axial agility, test case 4.

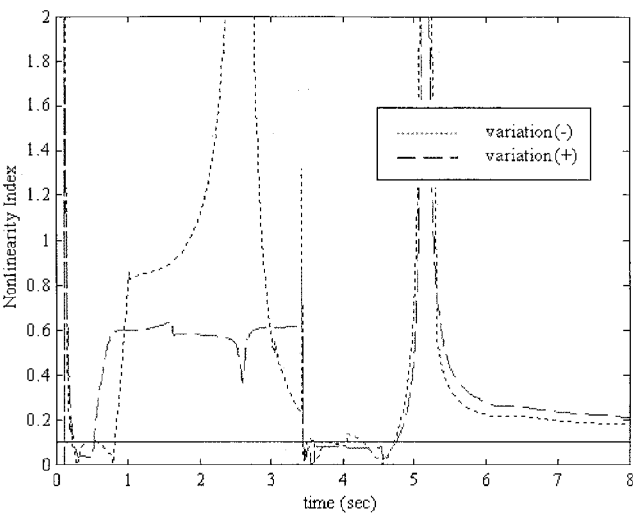

Fig. 15 Nonlinearity index for Cartesian power loss parameter metric, test case 4.

nearly linear to initial condition errors, especially when compared to the Cartesian coordinate version of the metric. When initial condition uncertainties and error propagation are considered, the Beck axial agility metric is judged to be less sensitive overall.

Conclusions

This paper investigated the relationship between initial condition error sensitivity and propagation to coordinate system for four nonpoststall agility metrics. A parameterization of Frenet-axis agility metrics was derived in terms of Cartesian body-axis forces, moments, and moments of inertia. Linear error theory was then used to perform an error propagation sensitivity analysis on each agility metric, using the same input data. Results were generated using a nonlinear, non-real-time, six-degree-of-freedom simulation of an UAV.

Based on the results presented in this paper, it is concluded that the accuracy of measured agility metrics is directly influenced by choice of coordinate system, according to the following:

1) To measure the lateral agility of an aircraft using the time to roll through bank angle metric, the Beck axial maneuver performance and axial agility metrics are both only weakly sensitive to initial condition errors, and these errors propagate almost linearly. The Beck curvature maneuver performance and curvature agility metrics are more sensitive and less linear, but by composition more appropriately quantify lateral maneuvers. However, for the same lateral maneuver, the time to roll through bank angle metric expressed in Cartesian coordinates is shown to be less sensitive and nonlinear overall than the Beck metrics, which are expressed in terms of the Frenet coordinate system.

2) For measuring pitch agility using the time-averaged integral of pitch rate, the Beck curvature maneuver performance metric was clearly less sensitive to initial condition errors and exhibited a more linear propagation of errors than all of the other Beck metrics tested. It was also less sensitive and more linear than the time-averaged integral of pitch rate metric expressed in Cartesian coordinates.

3) Results for the power onset parameter and power loss parameter were essentially the same, indicating that axial agility is best measured by the Beck axial agility and curvature agility metrics, which showed the least sensitivity to initial condition errors and a more linear propagation of errors. Both Beck metrics were also considerably less sensitive and more linear than their Cartesian coordinate counterparts. Overall, the Beck axial agility metric is judged to be less sensitive overall for quantifying axial agility than any of the other metrics tested.

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References

[1]Valasek, J., and Downing, D. R., "An Investigation of Fighter Aircraft Agility," NASA CR-1954608, Nov. 1993, p. 4.

[2]Liefer, R. K., Valasek, J., Eggold, D. P., and Downing, D. R., "Fighter Agility Metrics, Research, and Test," *Journal of Aircraft*, Vol. 29, No. 3, 1992, pp. 452–457.

[3]Bohacek, S., and Jonckheere, E., "Structural Stability of Linear Dynamically Varying (LDV) Controllers," *Proceedings of the 37th IEEE Conference on Decision and Control*, Vol. 4, IEEE, Washington, DC, 1998, pp. 4630–4635.

[4]Smith, D. M., and Valasek, J., "Agility Metric Robustness Using Linear Error Theory," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 2, 2001, pp. 340–351.

[5]Skow, A. M., and Hamilton, W. L., "Advanced Fighter Agility Metrics," Eidetics International, Technical Rept. 84-05, Hawthorne, CA, Sept. 1984.

[6]Beck, J. A., and Cord, T. J., "A Framework for Analysis of Aircraft Maneuverability," AIAA Paper 95-3448, Aug. 1995.

[7]McKeehen, P. D., and Cord, T. J., "Beck Maneuver Performance and Agility Metric Application to Simulation or Flight Data," AIAA Paper 98-4144, Aug. 1998.

[8]Junkins, J. L., "Adventures on the Interface of Dynamics and Control," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 6, 1997, pp. 1060–1063.

[9]Smith, D. M., "Agility Metric Sensitivity Using Linear Error Theory," M.S. Thesis, Aerospace Engineering Dept., Texas A&M Univ., College Station, TX, Aug. 2000, pp. 85–138.